

## ABSTRACT

Green's functions provide a simple, yet effective, method to test and calibrate General-Circulation-Model (GCM) parameterizations, to study and quantify model and data errors, to correct model biases and trends, and to blend estimates from different solutions and data products.

## Introduction

As part of ECCO (Estimating the Circulation and Climate of the Ocean, <http://www.ecco-group.org>, see also [1], [2], [3], and [4]), advanced assimilation methodologies (adjoint and Kalman filter) are being used to address the central objective of GODAE, the estimation of the time evolving, global-ocean circulation. Herein we use a much simpler approach, based on the computation of model Green's functions, to improve upon the existing ECCO circulation estimates. Specifically, model biases and trends are corrected and different solutions are combined.

## Green's Function Approach

Algebraically, the time-evolving ocean circulation can be written as a rule for time-stepping the oceanic state vector:

$$x(i+1) = \mathcal{M}[x(i), q(i)], \quad (1)$$

where  $x(i)$  is temperature, salinity, velocity, etc., on some predefined grid,  $\mathcal{M}$  represents the known time-stepping rules and boundary conditions, here an ocean GCM forced by the NCEP atmospheric reanalysis, and vector  $q(i)$  contains a set of unknown parameters and boundary conditions. The ocean data assimilation problem is the estimation of unknown parameters  $q$  and their uncertainty given a set of noisy measurements,

$$y = \mathcal{G}[q] + r, \quad (2)$$

Typically,  $q$  is estimated by minimizing a quadratic cost function of the data/representation error  $r$ , that is, the familiar least-squares minimization problem. Complications arise because the dimension of  $x$  is huge, because the covariance matrices of  $q$  and  $r$  are poorly known, and because  $\mathcal{G}$ , the function that relates unknowns  $q$  to the data  $y$ , is a non-trivial convolution of the ocean model  $\mathcal{M}$ . If the initial ocean circulation estimate is sufficiently realistic, then equation (2) can be linearized so that

$$y = Gq + r, \quad (3)$$

where  $G$  is a matrix whose columns are the Green's functions of model  $\mathcal{M}$  relative to measurements  $y$ . That is, each column of  $G$  can be computed as a GCM sensitivity experiment for the corresponding element in vector  $q$ .

## Data Assimilation Experiment

A Green's function optimization was carried out using the ECCO-2 [1] model configuration (75S to 75N, 1-deg horizontal grid spacing telescoping to 1/3-deg in the tropics, 46 vertical levels, KPP and GM-Redi mixing parameterizations). The period of assimilation is 1993-2000. The assimilated data are XBT, WOCE, PALACE, TAO, HOTs, and BATS temperature profiles (see Fig. 1). TOPEX/POSEIDON data are used for testing the solution. Table 1 lists the parameters, which were optimized. The parameters include global constants (mixing, surface relaxation and critical Richardson numbers) as well as spatially and/or temporally varying fields (isopycnal mixing, initial conditions, and surface fluxes) that were estimated as linear combinations of a small number of prior estimates obtained from satellite data products and from the ECCO adjoint-model and Kalman filter solutions.

# "Calibrating the ECCO Ocean General Circulation Model Using Green's Functions"

D.Menemenlis, I.Fukumori, T.Lee, and L.-L. Fu

Jet Propulsion Laboratory, California Institute of Technology  
Consortium for Estimating the Circulation and Climate of the Ocean (ECCO)

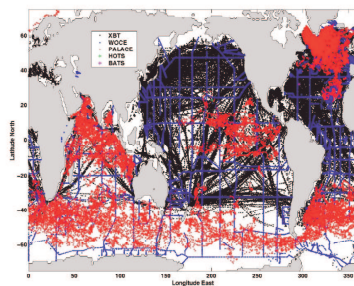


Fig. 1: Spatial distribution of data that were assimilated.

Table 1: Optimized control parameters before and after assimilation.

Parameter	Initial	Optimized
Vertical Diffusivity	$5 \times 10^{-4} \text{ m}^2/\text{s}$	$1.5 \times 10^{-5} \text{ m}^2/\text{s}$
Vertical Viscosity	$10^{-4} \text{ m}^2/\text{s}$	$1.9 \times 10^{-5} \text{ m}^2/\text{s}$
Ri (mixed layer)	0.3	0.332
Ri (shear instability)	0.7	0.704
Salt Relaxation	60 days	48 days
Temperature Relaxation	dQ/dT	1.44 dQ/dT
Isopycnal Mixing	500 $\text{m}^2/\text{s}$	linear combination
Initial Conditions	Spin-up	linear combination
Surface Fluxes	NCEP	linear combination

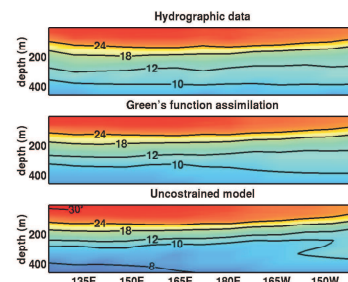


Fig. 2: Equatorial Pacific thermocline before and after assimilation.

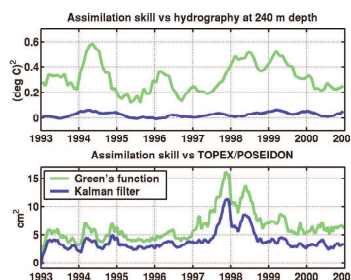


Fig. 3 : Globally-averaged assimilation skill.

## Results

The Green's function optimization substantially improves the time-mean stratification relative to the existing ECCO-2 solution; the cost function (weighted quadratic difference between data and assimilation results) is reduced by about 50%. This is largely due to a more realistic thermocline structure in the tropics. For example, Fig. 2 compares zonal slices of time-mean (1993-2000) temperature at 0N in the Pacific Ocean. Notice that the thermocline of the unconstrained solution is too sharp and too shallow relative to hydrographic data while the thermocline of the Green's function assimilation is much closer to the data. The improved representation of the thermocline largely results from the calibration of the vertical mixing coefficient; the realistic tropical thermocline structure is preserved even when the GCM is integrated for 40 years using the optimized parameters of Table 1.

Owing to the improved estimate of the time-mean state and to a combination of solutions, the Green's function optimization also improves model variability of temperature and sea-surface height relative to the data. Fig. 3 compares the assimilation skill (variance of simulation-data difference minus variance of assimilation-data difference) of the Green's function solution relative to that of the Kalman filter. Note that the skill of the Green's function solution improves upon the prior Kalman filter estimate throughout the period of analysis.

## Conclusions

We have demonstrated that the Green's function approach can be used to correct model biases and trends and to blend solutions from different prior estimates. The calibration of a small number of parameters is sufficient for improving many deficiencies of existing ECCO solutions. In particular the optimized mixing coefficients result in vastly improved thermocline structure and temperature variability. This is because the mixing parameters used in prior solutions (Kalman filter and adjoint) had been chosen using the traditional, one-at-a-time, trial-and-error approach, an approach that is both inefficient and sub-optimal: the optimal mixing parameters depend on each other and on the optimized surface fluxes and must therefore be estimated all-at-once.

We also find (not discussed hereinabove) that the Green's function approach is useful for testing and calibrating new GCM parameterizations and for the analysis of model and data errors.

## References

- [1] Fukumori, I., et al. *A Routine Global Ocean Data Assimilation System: ECCO-2*. "En route to GODAE" (this symposium).
- [2] Fukumori, I., et al. *On the Physical Consistency of Data Assimilated Estimates and the ECCO Hierarchical Assimilation Method*. "En route to GODAE" (this symposium).
- [3] Lee, T., et al. *Upper-ocean heat budget inferred from ECCO-2 ocean data assimilation system*. "En route to GODAE" (this symposium).
- [4] Stammer, D., et al. *Variations in the Large-Scale Ocean Circulation Estimated by Combining Satellite and WOCE Data With a Numerical Model*. "En route to GODAE" (this symposium).